Furthermore, their ordinate intersections are function of σ_0/σ_c and in particular, of

$$Z_o = \frac{1}{\sqrt{3}}\left(p_i + p_l + p_e\right) \qquad (13)$$

This signifies that, contrary to the criterion of Von Mises, the extent of the elastic domain in the present case is no longer independent of the hydrostatic component of the load vector; the elastic domain enlarges as Z increase, the lines forming the contour remaining parallel to themselves. Moreover, it should be noted that, with the system of axes V, W, Z, the V-axis coincides with the projection p_l' of p_l on the plane π.

The graphic method described in the preceding paragraph is also valid in this case. It is slightly complicated because the dimensions of the elastic zone

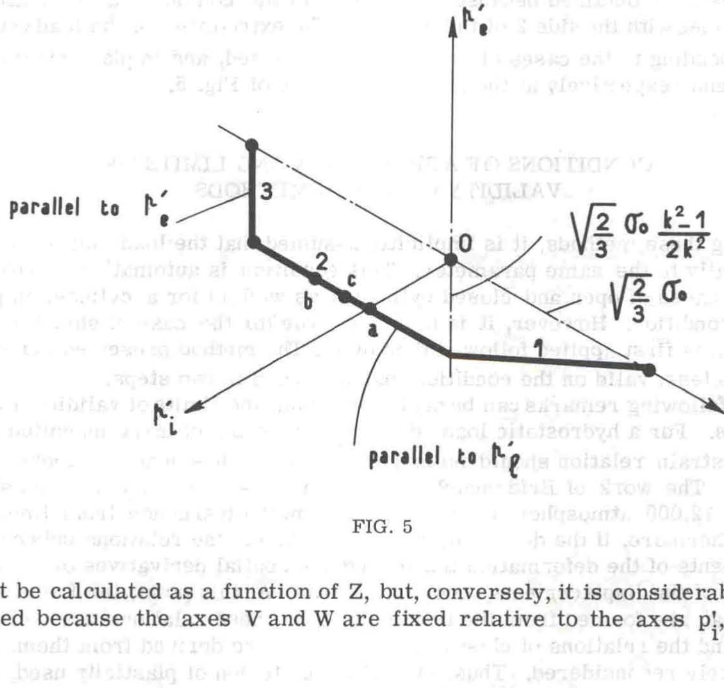

FIG. 5

must be calculated as a function of Z, but, conversely, it is considerably simplified because the axes V and W are fixed relative to the axes p_i', p_l', p_e'.

ELASTIC LOADING FOR TRESCA'S CRITERION

Letting $\sigma_0 = \sigma_c$, the oblique lines of the precedent intrinsic curve become parallel and the criterion of Mohr-Caquot reduces to that of Tresca. In the space p_i, p_l, p_e, the criterion of Tresca is represented by an irregular hexagonal prism inscribed in the elliptic cylinder of Von Mises. The magnitude of the elastic domain is once again, as in the case of Von Mises, independent of the hydrostatic component of the load vector. The intersection of this prism with the plane π gives the contour of the elastic domain. Fig. 5 shows that this contour can be easily traced, the other half of the hexagon being symmetric with respect to the origin. For $k = 3$, side 1 of the hexagon almost coincides with the perpendicular to p'_e (the position of 1 in the figure corresponding to $k = \infty$). This shows that increasing k above the value 3 adds only a small gain to the elastic loading. For $k = 1$, the hexagon is reduced to the line p'_1 (zero surface).

The study of the maxima uncovers the following well-known result: For a given p_e there exists, contrary to the criterion of Von Mises, an infinite number of values of p_l for which p_i is maximum and equal to

$$\frac{\sigma_0}{2}\left(1 - \frac{1}{k^2}\right)$$

This result is obtained because the tangent of the contour drawn parallel to p'_1 coincides with the side 2 of the hexagon. The extremities of the load vectors corresponding to the cases of cylinders open, closed, and in plane strain condition, end respectively at the points a, b, and c of Fig. 5.

CONDITIONS OF APPLICATION AND LIMITS OF VALIDITY OF THESE METHODS

Using these methods, it is implicitly assumed that the loads increase proportionally to the same parameter. This condition is automatically satisfied for the cases of open and closed cylinders as well as for a cylinder in plane strain condition. However, it is no longer true for the case of shrink fits, in which p_e is first applied followed then by p_i. The method presented herein is, nevertheless, valid on the condition that it is used in two steps.

The following remarks can be made concerning the limits of validity of these methods. For a hydrostatic load, $P_h = p_i = p_l = p_e$, of large magnitude, the stress-strain relation should no longer be linear, thus making Hooke's law invalid. The work of Bridgman[8] on the compressibility of pure iron shows that, at 12,000 atmospheres, there exists small divergence from linearity.

Furthermore, if the deformations become large, the relations between the components of the deformation tensor and the spatial derivatives of the components of the displacements become quadratic. At this point, the Lamé equations that are formed from the linear forms at these relations are no longer valid, and the relations of elastic loading, which are derived from them, must be entirely reconsidered. Thus, even if the criterion of plasticity used, as in the case for the criteria of Von Mises, Mohr-Caquot and Tresca, implies the condition that a hydrostatic constraint does not cause plastic deformation, it does not automatically result that a hydrostatic load, $p_h = p_i = p_l = p_e$, protects the cylinder from all plastic flow.

Conversely, if the loads, though large, are not isotropic ($p_i \neq p_l \neq p_e$) it can be considered that a plastic law governs the deformation beyond the elastic

[8] Bridgman, P. W., "The Physics of High Pressure," 2nd Edition, Bell and Sons, London, England, 1949, p. 154.